33rd ALERT Workshop – Poster Session

4-cell analogical model to describe plastic shear behavior of granular geomaterials

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Quasi-static Elasto-plastic models of Granular Solid

* Phenomenological EP model **(Cam-clay (Schofield & Wroth 1968))**

Too many ad-hoc models, too many parameters

→**Physically-based constitutive model** should be explored

* Micromechanics models

Regular periodic packing models

(Newland & Allely 1957, Rowe 1962, Matsushima & Chang 2011)

Random packing averaging models

Elastic models **(Digby 1981, Christoffesen, et al. 1981, Walton 1987, Bathurst & Rothenburg 1988, Chang & Misra 1990, etc.)**

EP models **(Chang et al. 1992, Chang & Hicher 2005, Matsushima & Chang 2007, etc.)**

Cell-based models **(Nicot and Darve 2011, etc.)**

Objective of this study

The present model is:

Modification of **Matsushima & Chang model (2007)** inspired by

Regular packing models

Cell-based models

In this presentation, I will explain:

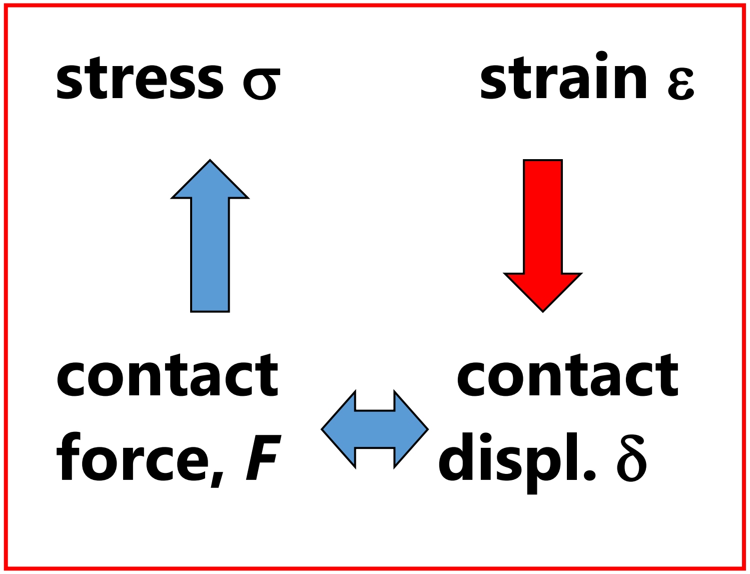
(1) Matsushima & Chang model (2007)

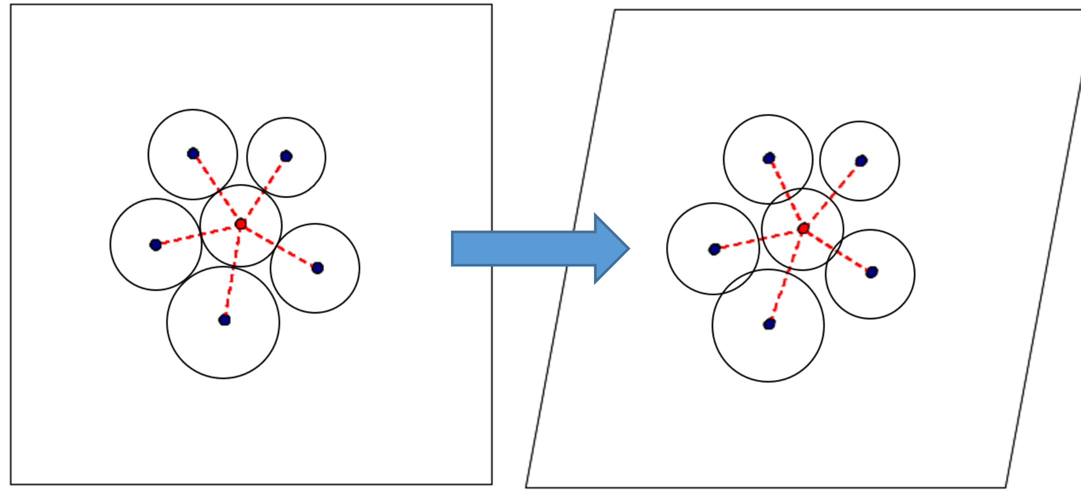
Uniform strain model with finite deformation formulation

(2) **4-cell analogical model** to control contact normal force distribution

(3) Biaxial test (const. confining pressure/ const. volume) responses

Uniform strain model (Chang & Misra 1990)





Assumption:
Contact displacement of each branch is uniquely determined by the bulk strain field

Particle centroids move sticking to the continuum strain field

grain rotation = continuum rotation

→contact displ. δ

Finite deformation formulation

(Matsushima & Chang 2007)

$$\boldsymbol{F}_t(\tau) = \boldsymbol{I} + d\boldsymbol{\varepsilon}_t$$

Strain increment from time t to τ

$$\boldsymbol{F}_t(\tau) = \boldsymbol{V}_t(\tau)\cdot\boldsymbol{Q}_t(\tau)$$

$$\boldsymbol{l}_\tau^{ref} = \boldsymbol{Q}_t(\tau)\cdot\boldsymbol{l}_t^{ref}$$

Branch vector of an unloaded state

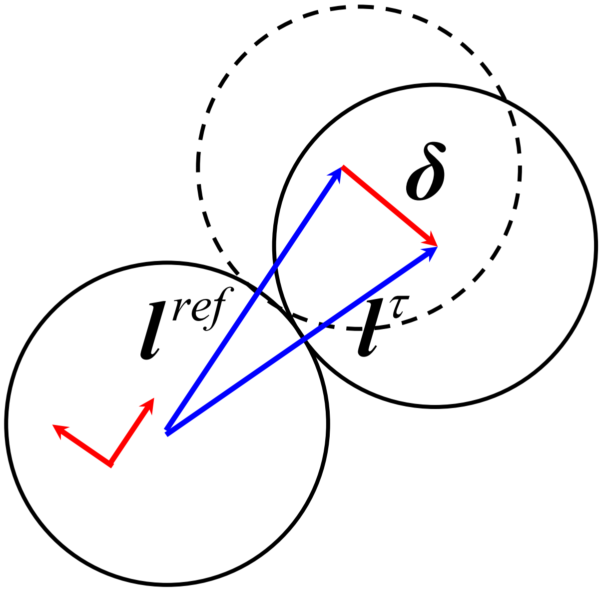
$$\boldsymbol{l}^\tau = \boldsymbol{F}_t(\tau)\cdot\boldsymbol{l}^t$$

$$\boldsymbol{\delta} = \boldsymbol{l}^\tau - \boldsymbol{l}_\tau^{ref}$$

$$\boldsymbol{\delta}^L = \boldsymbol{R}^T\cdot\boldsymbol{\delta}$$

$$\boldsymbol{f}^L = \begin{pmatrix} f_n \\ f_s \end{pmatrix} = \begin{pmatrix} k_n & 0 \\ 0 & k_s \end{pmatrix}\cdot\boldsymbol{\delta}^L$$

$$\boldsymbol{f} = \boldsymbol{R}\cdot\boldsymbol{f}^L$$



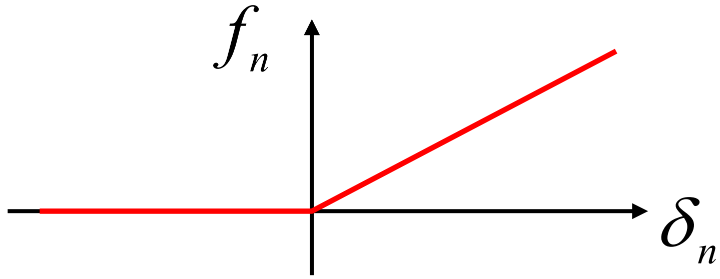
$$\boldsymbol{\sigma}^{tT} = \frac{1}{V_R^t}\sum_c^N(\boldsymbol{l}^t\otimes\boldsymbol{f}^t)$$

Love-Weber equation

Contact loss & sliding (Matsushima & Chang 2007)

Contact loss (nonlinear elasticity)

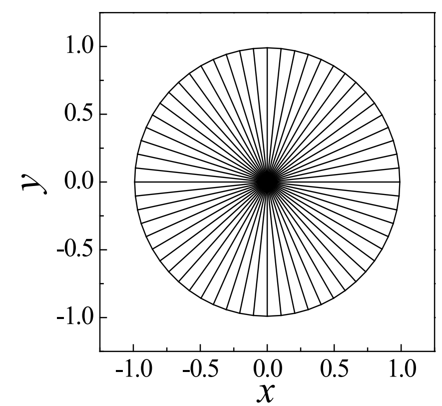
No tensile contact force



Contact **sliding** (elasto-plasticity)

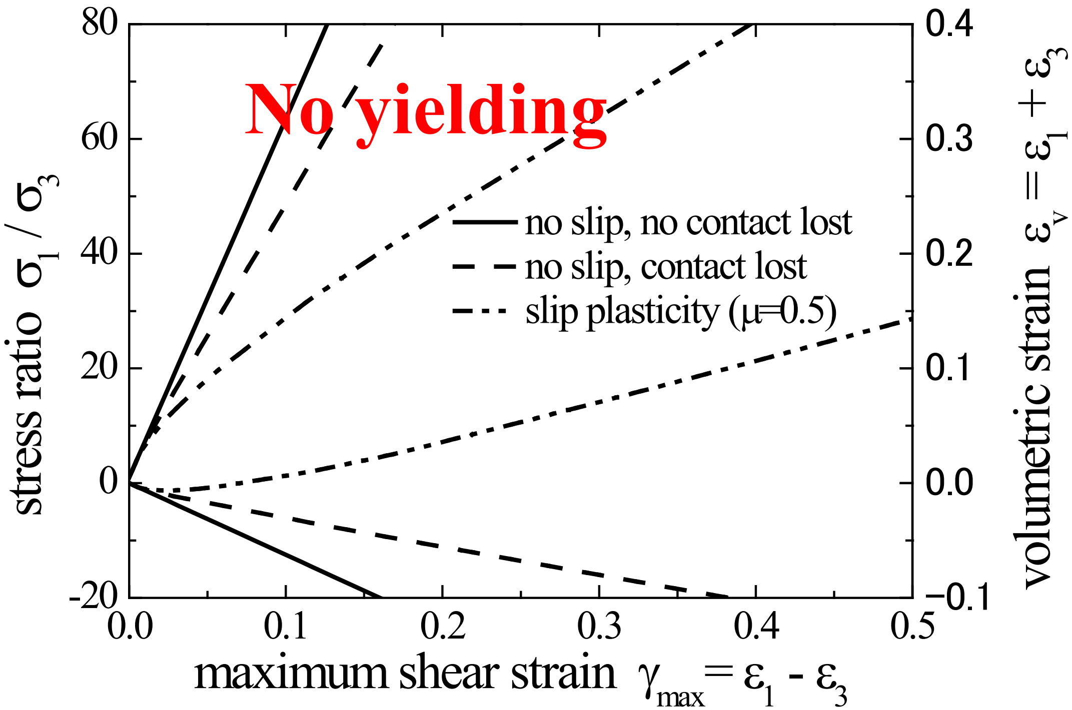
$$|f_s| \begin{cases} \le -\mu f_n & \rightarrow (elastic) \\ > -\mu f_n & \rightarrow (sliding) \rightarrow |f_s| = -\mu f_n sign(f_s) \end{cases}$$

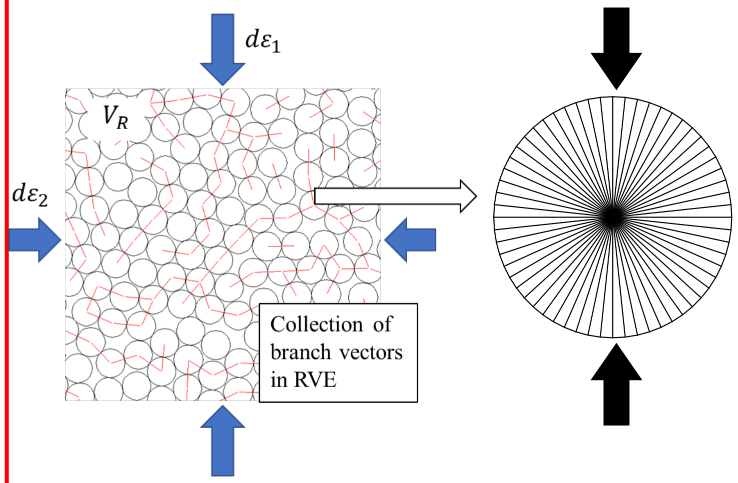
A sufficient number of **branch vectors** are **explicitly** assigned in the program
→ Compute the response



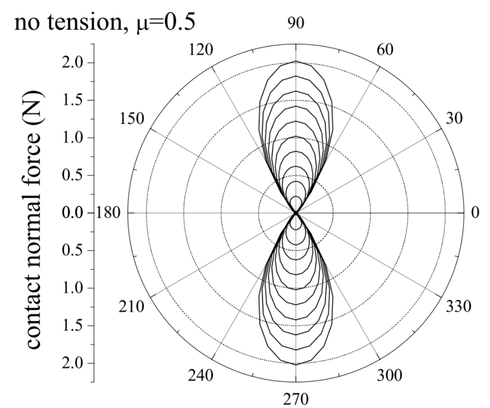
Example of the response (Matsushima & Chang 2007)

Biaxial test result





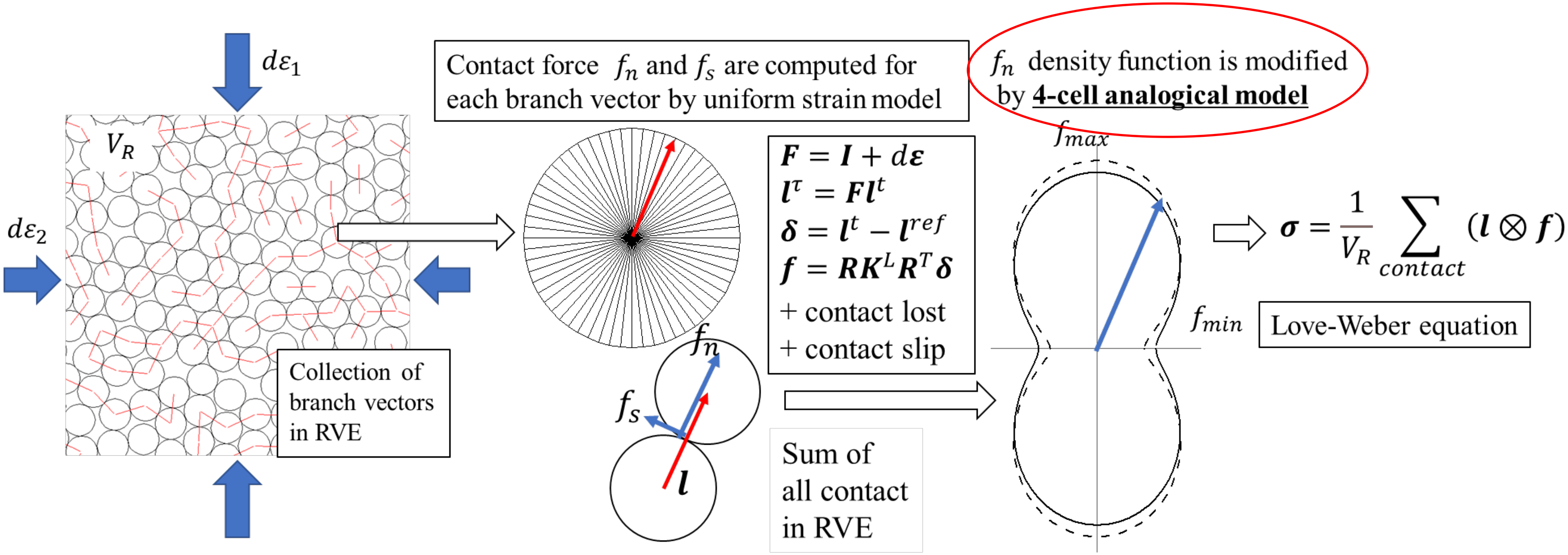
Branch vectors along the principal stress direction do not slide.



Contact normal force distribution does not converge.

Disadvantage of uniform strain model

Proposed model (4-cell analogical model)



f_n (normal contact force) distribution function is modified by **4-cell analogical model**

4-cell analogical model (1)

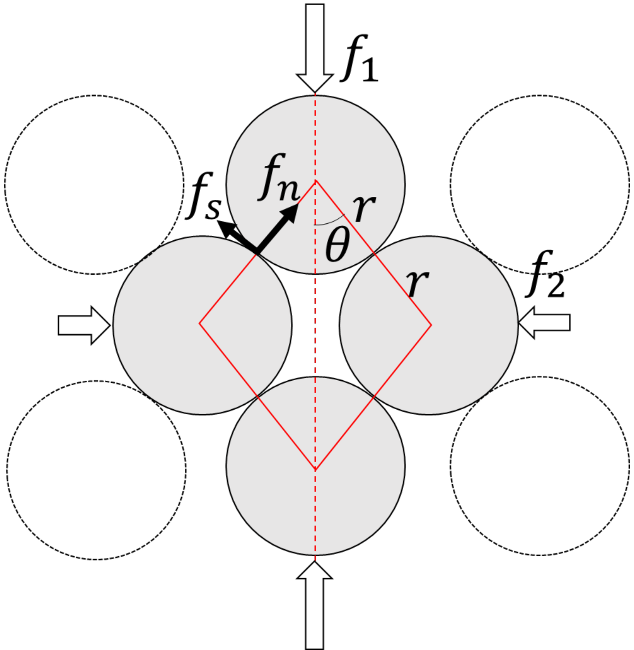
First, we consider **2D regular periodic packing** in the figure.

Relation between the **porosity *n*** and a **structural state parameter θ** is:

$$n = \frac{\pi}{4\sin 2\theta} \qquad (30^\circ \leq \theta \leq 45^\circ \text{ to avoid the instable response})$$

The internal contact normal and tangential force in the hatched 4 cell is f_n and f_s

f_1 and f_2 are the sum of the external force acting on the 4 cell structure in the horizontal and the vertical direction, respectively.



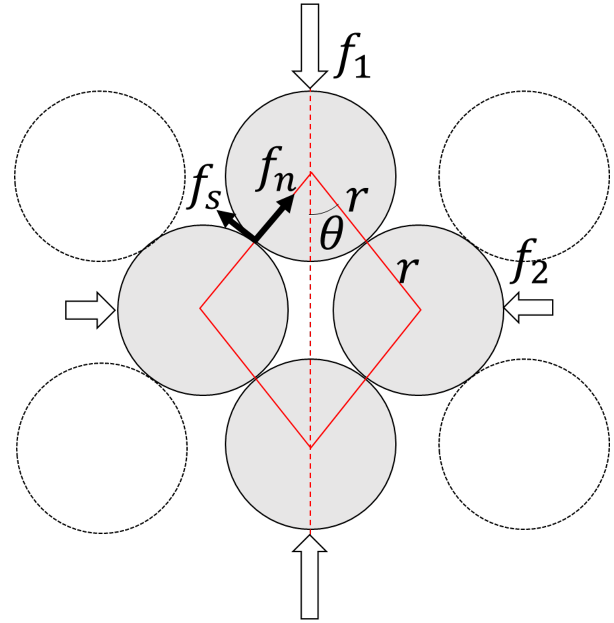
Then the **equilibrium of the top particle** is described by

$$\frac{1}{2}f_1 = f_n\cos\theta + f_s\sin\theta$$

$$\frac{1}{2}f_2 = f_n\sin\theta - f_s\cos\theta$$

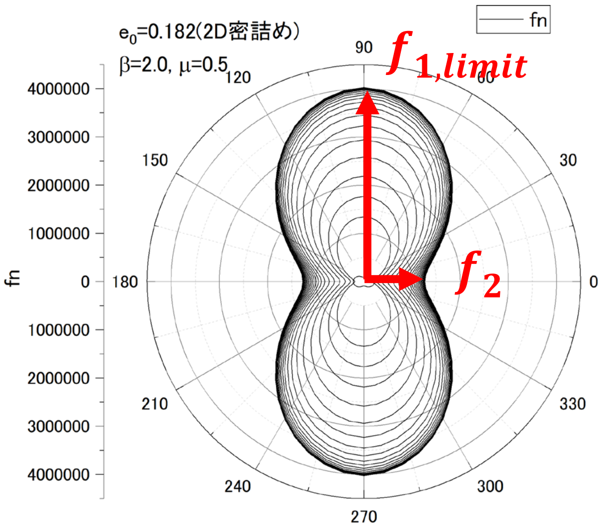
4-cell analogical model (2)

$$\frac{1}{2}f_1 = f_n \cos\theta + f_s \sin\theta$$
$$\frac{1}{2}f_2 = f_n \sin\theta - f_s \cos\theta$$



These equations together with the equation of the contact slip criterion, $f_s = \hat{c} + \hat{\mu}\, f_n$, we obtain the following equation:

$$f_{1,limit} = \frac{2\hat{c} + f_2(1 + \hat{\mu}\tan\theta)}{\tan\theta - \hat{\mu}}$$



In this presentation, $\hat{c} = 0$ is assumed for simplicity.
Then

$$\alpha = \frac{f_{1,limit}}{f_2} = \frac{1+\hat{\mu}\tan\theta}{\tan\theta-\hat{\mu}}$$

is the **critical aspect ratio** of the orientational distribution function of f_n.

4-cell analogical model (3)

Computational flow:

(1) Compute $f_{n,max}$ and $f_{n,min}$ from uniform strain model.

(2) Compute $f_{1,limit} = \frac{f_{n,min}(1+\hat{\mu}\tan\theta)}{\tan\theta-\hat{\mu}}$

If $f_{1,limit} < f_{n,max}$

$$\Delta f = \frac{f_{n,max} - \alpha f_{n,min}}{1 + \alpha\beta}$$

$$f_{2,lim} = f_{n,min} + \beta\,\Delta f, \qquad f_{1,lim} = f_{n,max} - \Delta f$$

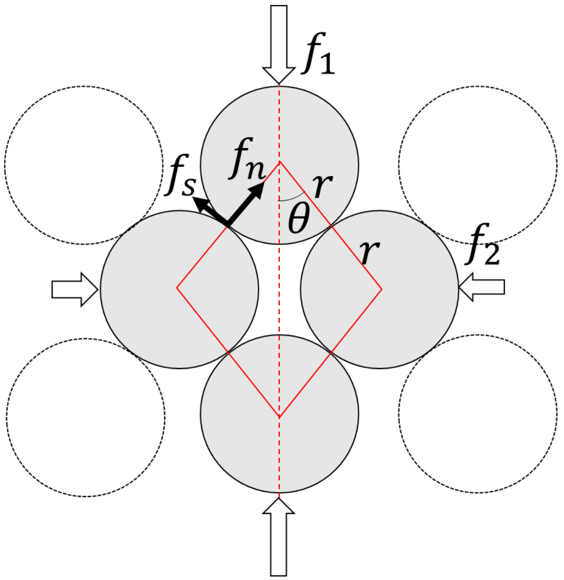
where β is a **parameter to control dilation**

and model as a function of the structural parameter θ as:

$$\beta = \beta_0 \frac{\theta_{max} - \theta}{\theta_{max} - \theta_{min}}$$

$$\theta_{max} = \pi/4, \quad \theta_{min} = \pi/6$$

$\theta = \theta_{max} \rightarrow \beta = 0 \rightarrow f_{1,lim} = \alpha f_{n,min}, f_{2,lim} = f_{n,min}$ (**critical state**)

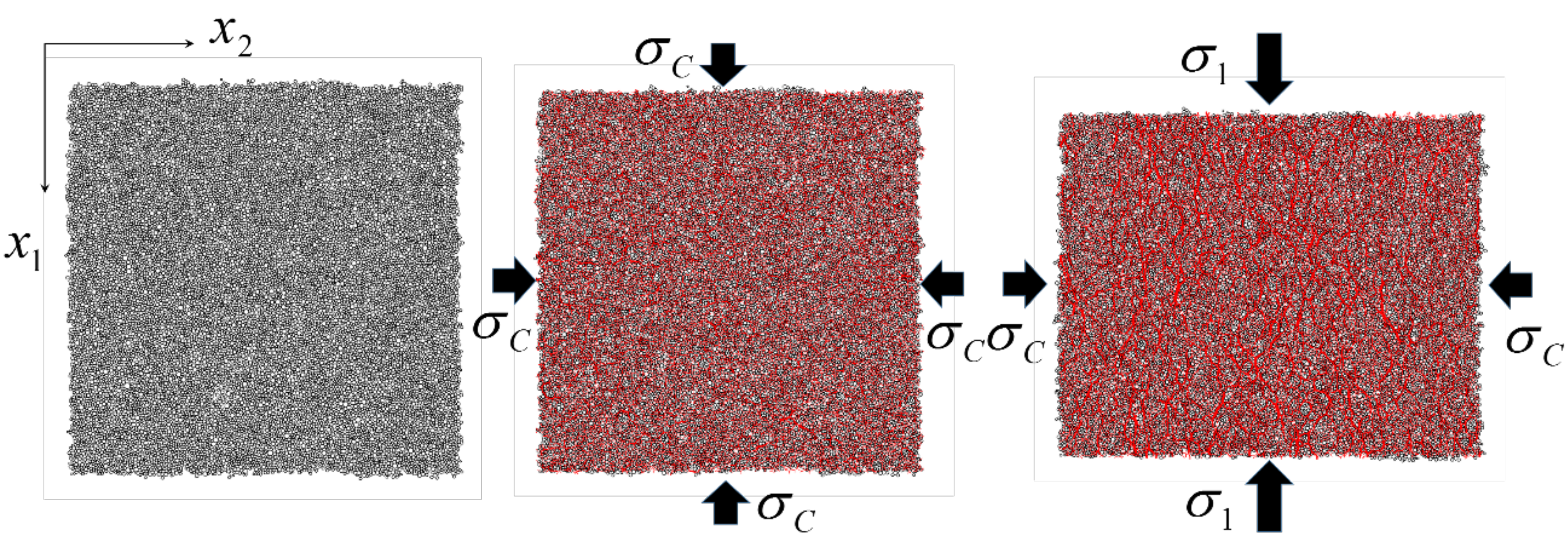


Biaxial Test Response

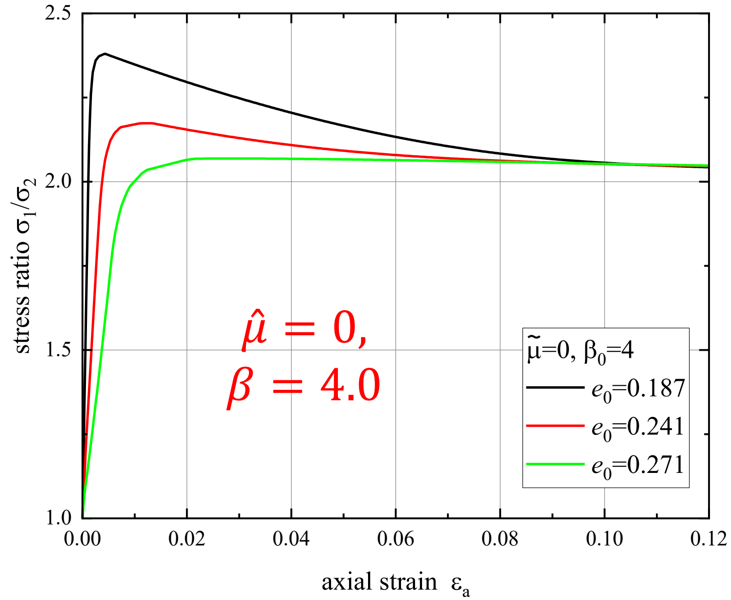
Grain size	0.1(mm)
Contact spring constant $k_n = k_s$	1.0×10^9 (N/m)
Intergranular friction μ	0.5
$\hat{\mu}$	0.0, 0.5
β_0	2.0, 4.0
Confining pressure σ_c	100(kN/m)
Strain increment $\Delta\varepsilon_1$	5.0×10^{-5}

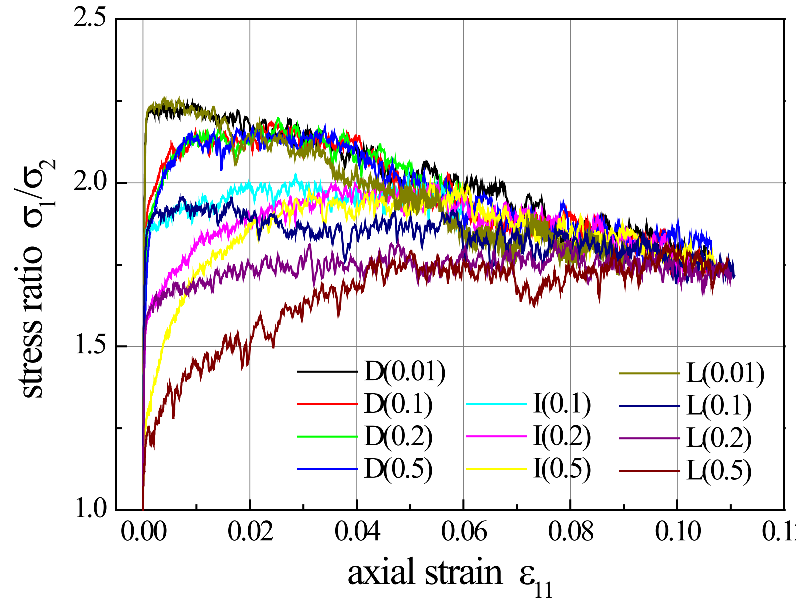
The results are compared with DEM simulation (Matsushima 2015):

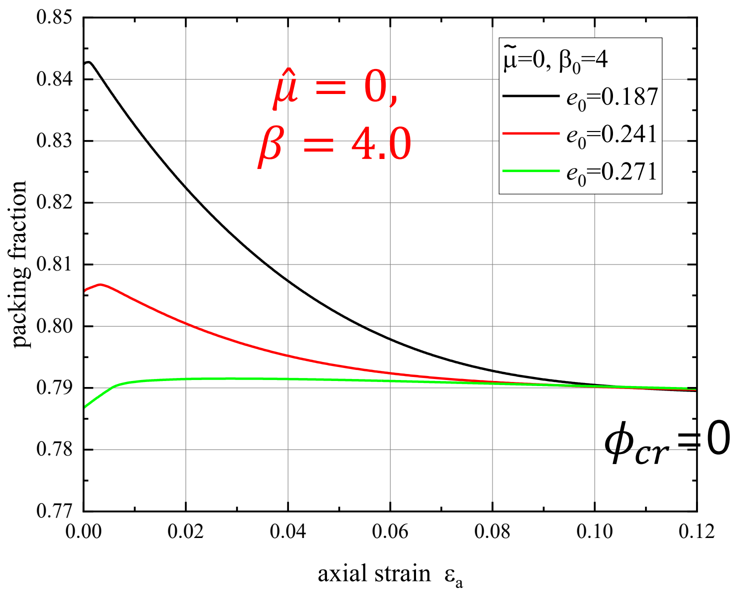
Biaxial compression with double periodic boundary
20,000 circular grains
Starting from various initial void ratio



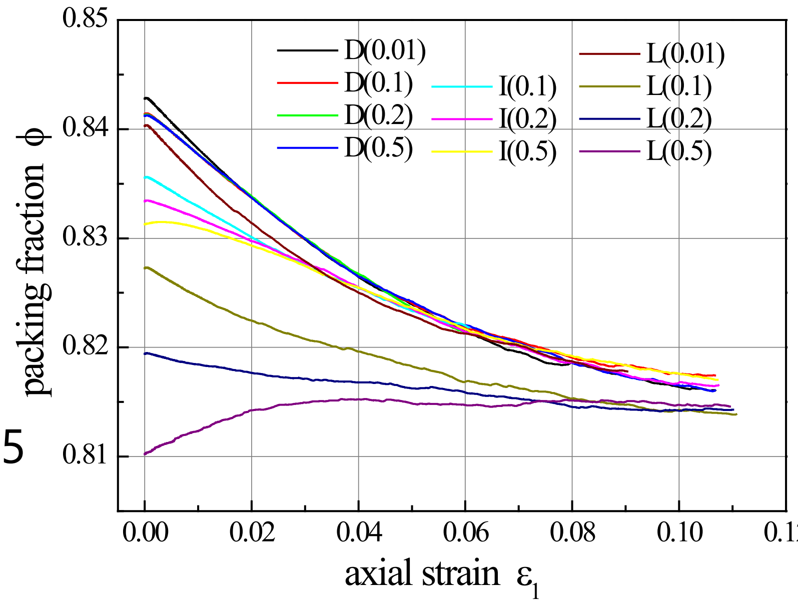
Biaxial Response (const. confining pressure)





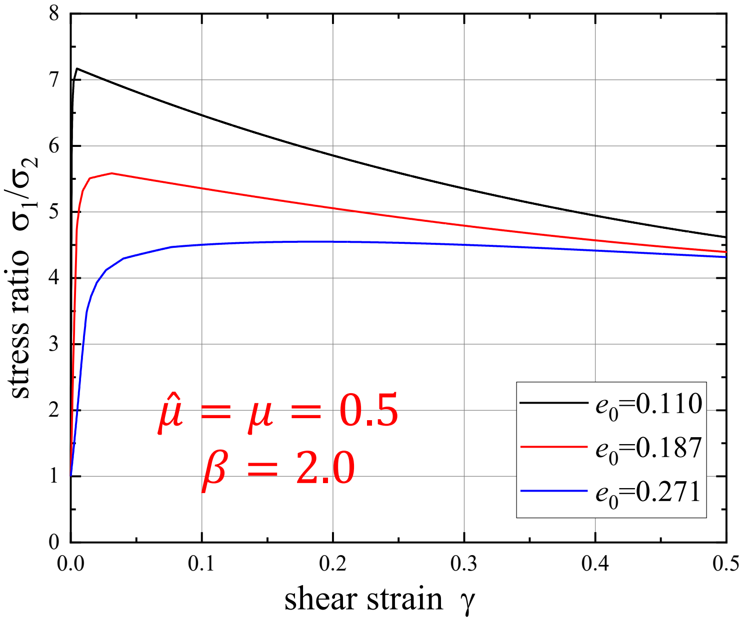


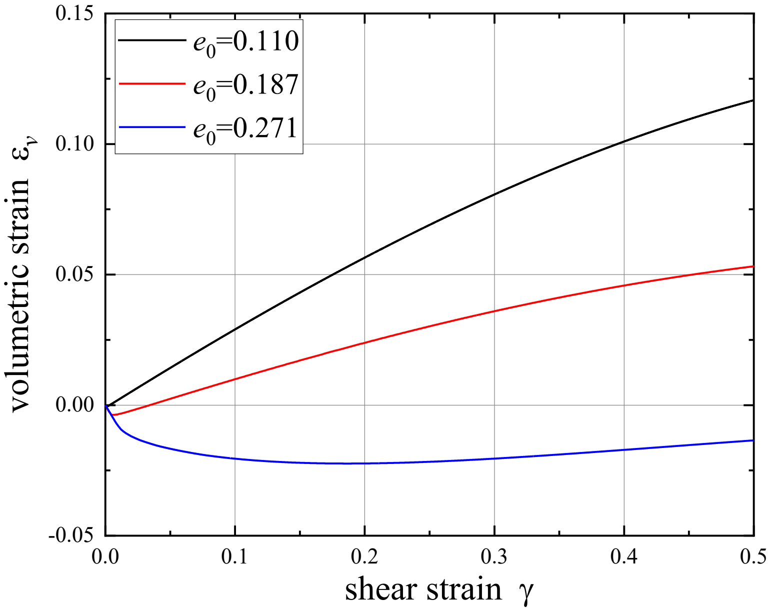
ϕ_{cr}=0.7895



ϕ_{cr}=0.815
e_{cr}=0.2270

Biaxial Response (const. confining pressure)



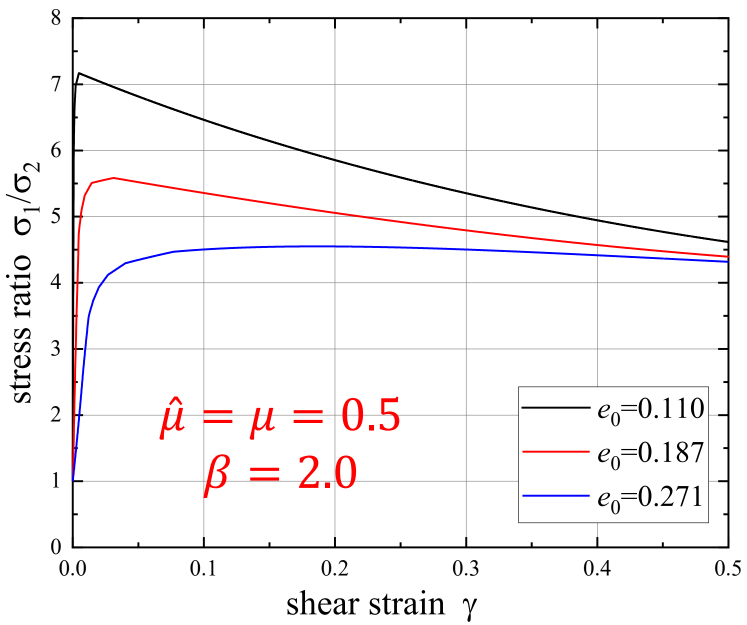


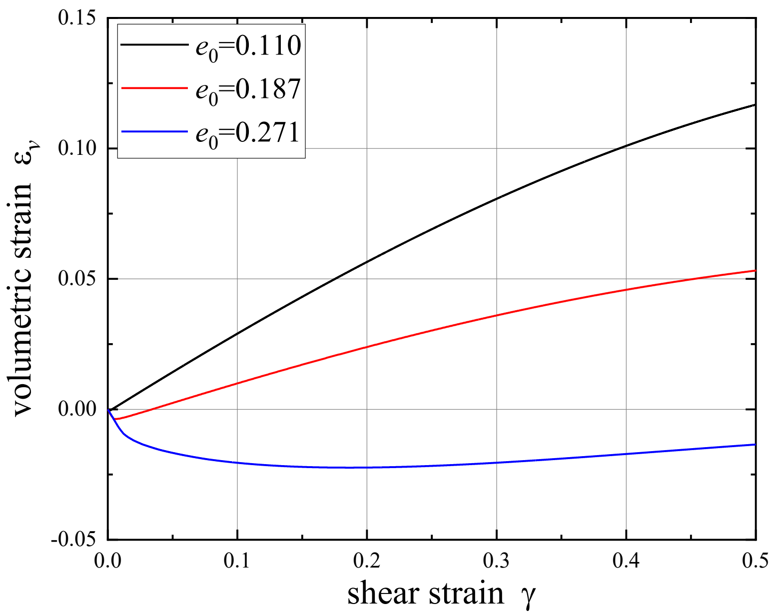
Material with different initial void ratio reaches the same critical state.
The volumetric strain also converges smoothly.

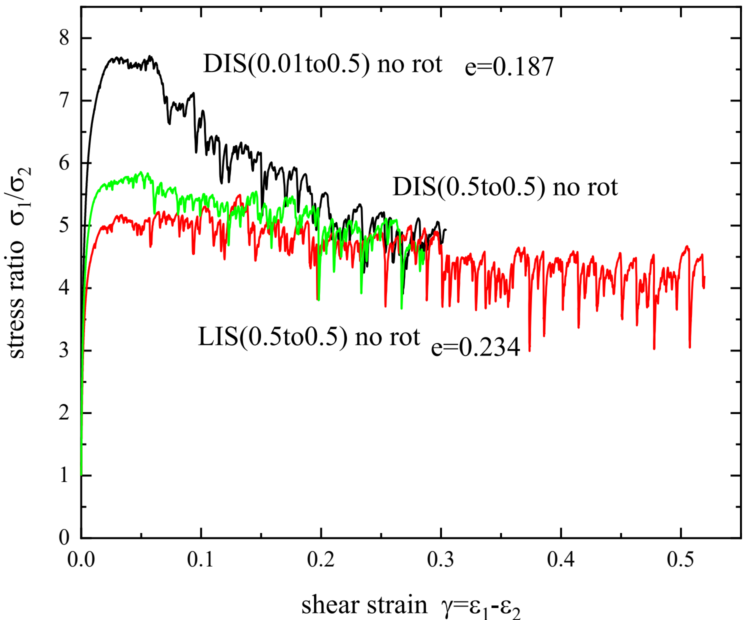
$\hat{\mu}$ controls the shear strength

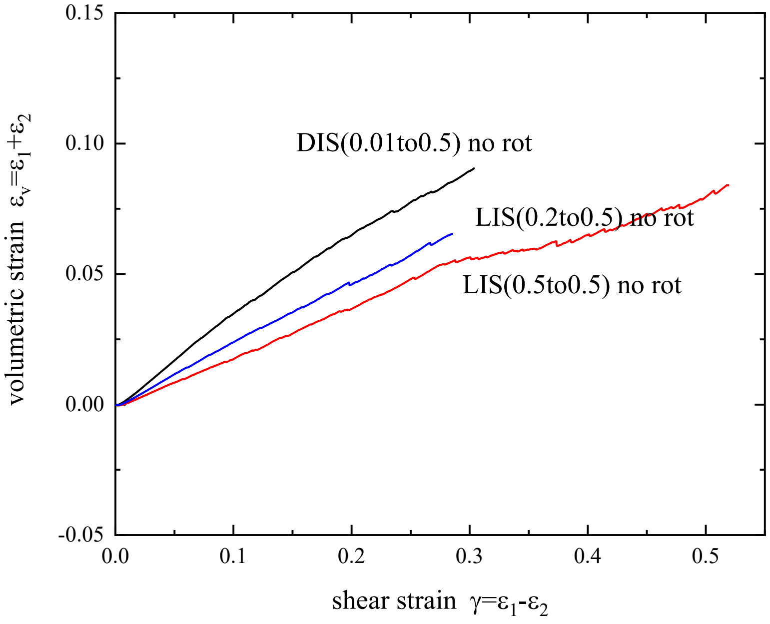
β controls how fast the material reaches the critical state

Biaxial Response (const. confining pressure)



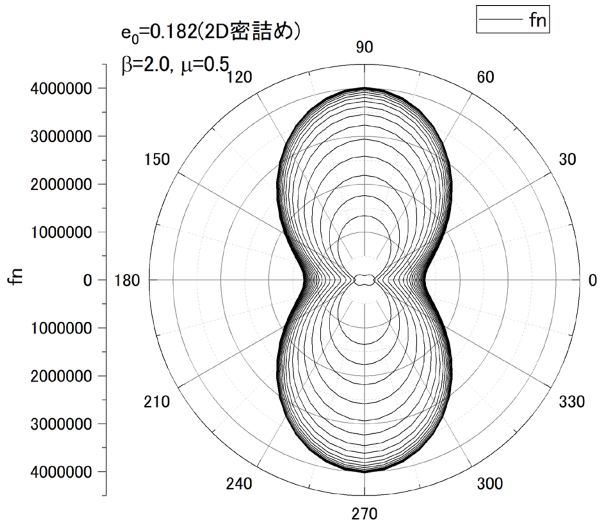


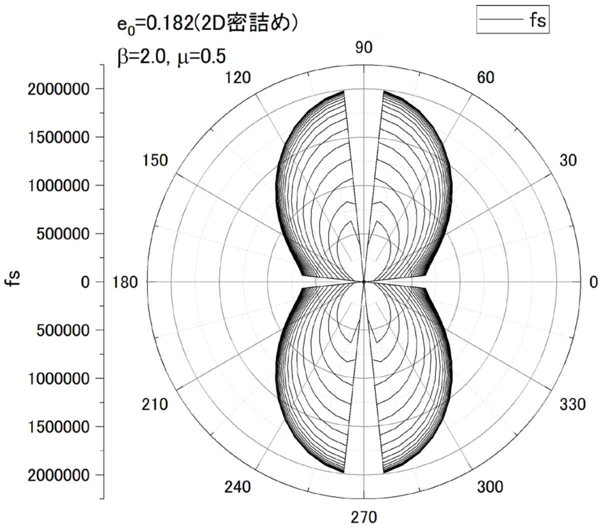


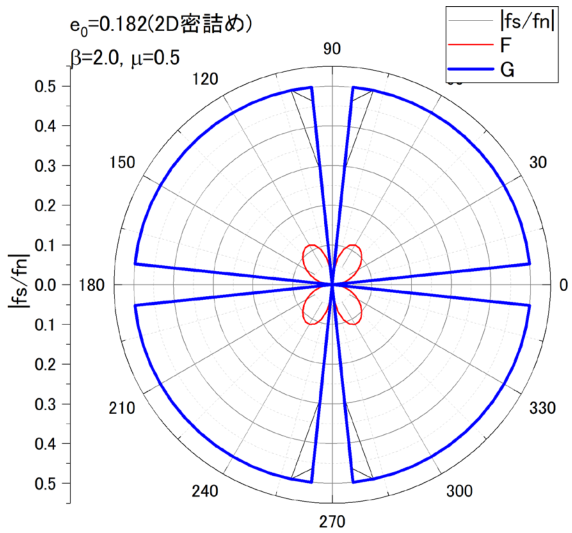


Biaxial Response (const. confining pressure)

$\hat{\mu} = \mu = 0.5$
$\beta = 2.0$

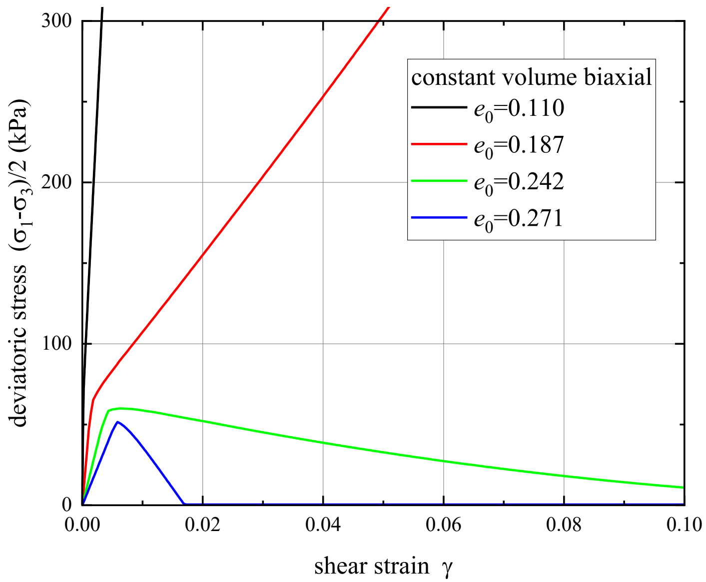


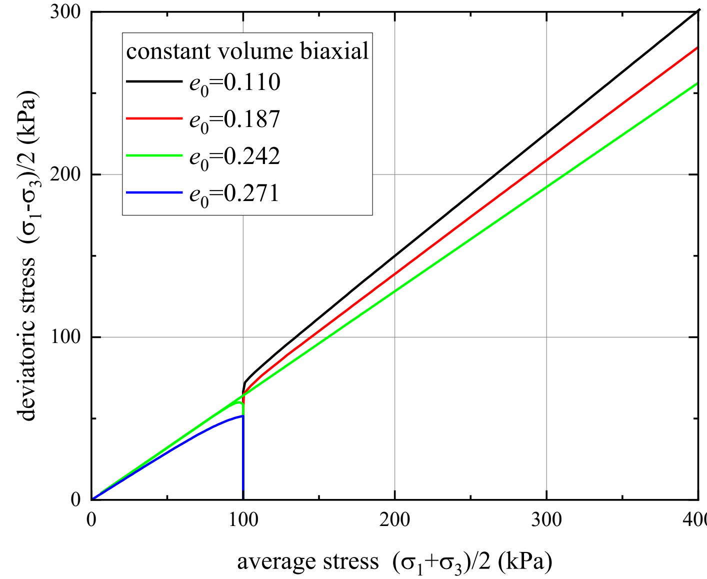




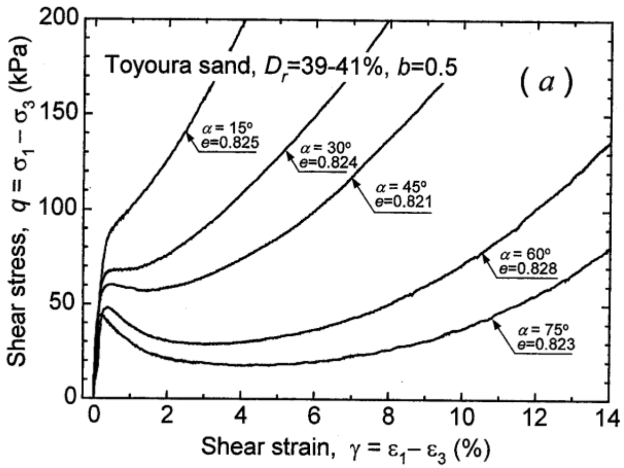
Distribution of f_n and f_s is converged at the critical state.
All the contacts except principal stress direction is sliding at the critical state.

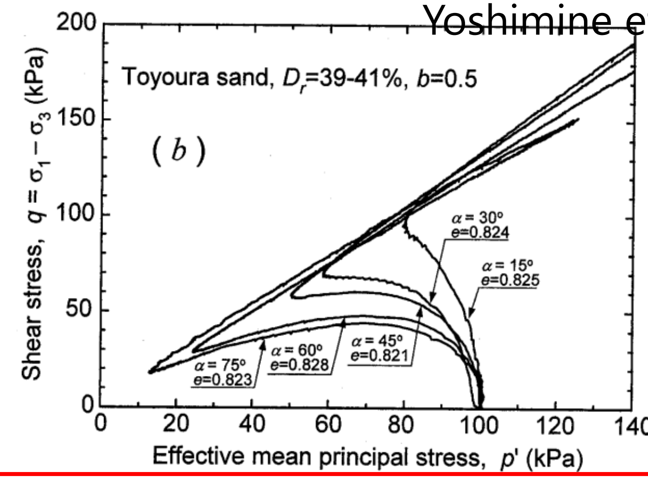
Biaxial Response (const. volume)





Yoshimine et al. 1998





Constant volume test (undrained test) also provides reasonable response.
Transition from elastic to plastic regime will be smoother
if initial tangential contact force is imposed in each branch.

Conclusions

A new micromechanics model based on uniform strain model is proposed.
The model consists of

relation between void ratio (porosity) and structural parameter

model to control the critical f_n distribution

Basic material parameters are

* contact stiffness and friction coefficient

Only two additional parameters to control

* aspect ratio of the critical f_n distribution (shear strength)
* evolution of dilation

3D model will be straightforward.
→Comparison with experiment is ongoing.

References

Chang, C.S. and Misra, A., 1990. Journal of engineering mechanics, 116(10), pp.2310-2328.

Matsushima, T. and Chang, C.S., 2007. In Geomechanics and Geotechnics of Particulate Media, pp. 293-298, CRC Press.